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INVESTIGATION OF PLANE STRAIN VIBRATIONS IN THERMOPOROELASTIC PLATES

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Abstract:- This paper deals with the plane strain vibrations in thermoporoelastic plates in the framework of Biot's theory. Pertinent constitutive relations and governing equations are derived. Frequency equation is obtained in the presence of dissipation. In the particular case the frequency equation is obtained in the absence of dissipation. Frequency and attenuation is computed as a function of wavenumber. For illustration purpose, three poroelastic solids namely sandstone and berea sandstone are employed.

Keywords: *Thermoporoelastc plate, Frequency equation, Wavenumber, Frequency, Attenuation*

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I INTRODUCTION

The study of wave propagation in thermoporoelastic plates has wide applications in many fields such as Engineering, Biomechanics and Geophysics. The soft tissues in biological bodies are treated as a thermoporoelastic media. Nowacki and Sokolowski [1] studied propagation of thermoelastic waves in plates. Thermoelastic waves in thin plates is investigated by Massalas [2]. In the said paper, a mathematical analysis is presented to study the wave characteristic of the plate and special cases of very short and very long waves are discussed. The propagation of plane waves in an infinite homogeneous isotropic thermoelastic plate is studied in the context of coupled theory of thermoelasticity Sharma et al [3]. The effects of plate thickness on the symmetric and skew symmetric modes of vibrations are discussed. Generalized thermoelastic waves in homogeneous isotropic plates is studied by Sharma et al [4]. Valering Salnikov and Nigel Scott [5] studied thermoelastic waves in a constrained isotropic plate. In the said paper, the boundaries of the plate are taken to be traction free and isothermal or insulated. Dispersion relations are derived and expanded asympotically in the long wave low frequency limit. Damping of generalized thermoelastic waves in a homogeneous isotropic plate is studied by Selvamani and Ponnuswamy [6]. In the said paper, the frequency equation is obtained by the traction free boundary conditions using the Bessel function solutions. K.L.Verma [7] investigated on the dynamic characteristic of thermoelastic waves in thermoelastic plates with thermal relaxation time. The exact solutions of the displacement, temperature and thermal stresses, temperature gradient in an infinite plate of arbitrary anisotropy of finite thickness are derived for the generalized theory of thermoelasticity with two relaxation times. Employing the Biot's theory [8], Dispersion study of plane strain vibrations in poroelastic solids bars with polygonal cross section studied by Sandhya Rani et al [9]. In this paper, Fourier collocation method is used to solve the plane strain problem. The frequency equation in the case of symmetric and anti-symmetric modes are discussed. The phase velocity is computed as a function of wavenumber in the cases of triangular, square, pentagon, and hexagon cross sectional cylinders. Paul and Murali [10] studied wave propagation in the thermoporoelastic plate. In the paper [10], governing equations of thermoporoelastic plate are derived and the frequency equation is obtained for stress free and thermally insulated boundary conditions. Wave propagation in thermoelastic saturated porous medium is investigated by M.D.Sharma [11]. In this paper, three longitudinal and one transverse waves in an isotropic thermoelastic porous solid are obtained. Relations are derived among the temperature of the medium and the displacements of fluid and solid particles. However to the best of author's knowledge plane strain vibrations of thermoporoelastic plates are investigated. Frequency and attenuation is computed as a function of wavenumber, for the three types of thermoporoelastic materials and then discussed

II. GOVERNING EQUATIONS AND SOLUTION OF THE PROBLEM

The dynamic equations in cartesian coordinate system in the absence of body forces [8] and heat conduction are as follows

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+\frac{\partial \sigma_{xz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U)+b\frac{\partial}{\partial t}(u-U),$$

$$\frac{\partial \sigma_{yx}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{yz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V)+b\frac{\partial}{\partial t}(v-V),$$

$$\frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W)+b\frac{\partial}{\partial t}(w-W),$$

$$\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U)-b\frac{\partial}{\partial t}(u-U),$$

$$\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V)-b\frac{\partial}{\partial t}(v-V),$$

$$\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w+\rho_{22}W)-b\frac{\partial}{\partial t}(w-W),$$

$$K\nabla^2 T=\rho c_v(T+\tau_0\frac{\partial T}{\partial t})\frac{\partial}{\partial t}+\beta T_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla.u$$

(1)

In eq. (1), $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, ρ is the mass density, c_v is the specific heat capacity, K is the thermal conductivity, T_0 is the reference temperature, τ_0 is the relaxation time, $\rho_{11},\rho_{12},\rho_{22}$ are the mass coefficients, (u,v,w) and (U,V,W) are the displacements of solid and fluid. s is the fluid pressure, b is the dissipation coefficient, σ_{ij} are the stress components are given by [12]

$$\sigma_{xx}=2Ne_{xx}+Ae+Q\varepsilon-\beta(T),$$

$$\sigma_{yy}=2Ne_{yy}+Ae+Q\varepsilon-\beta(T),$$

$$\sigma_{zz}=2Ne_{zz}+Ae+Q\varepsilon-\beta(T),$$

$$s=Qe+R\varepsilon.$$

(2)

In eq. (2), e_{ij}'s are strain components, A, N, Q, R are poroelastic constants, β is the thermal stress, T is the temperature, e and ε are dilatations of solid and fluid. Substitution ofeq. (2) in eq. (1) the equations of motion for the plane strain problem are as follows

$$N\nabla^2 u + (A+N)\frac{\partial e}{\partial x} + Q\frac{\partial \varepsilon}{\partial x} - \beta\frac{\partial T}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U) + b\frac{\partial}{\partial t}(u-U),$$

$$N\nabla^2 w + (A+N)\frac{\partial e}{\partial z} + Q\frac{\partial \varepsilon}{\partial z} - \beta\frac{\partial T}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}w + \rho_{12}W) + b\frac{\partial}{\partial t}(w-W),$$

$$Q\frac{\partial e}{\partial x} + R\frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U) - b\frac{\partial}{\partial t}(u-U),$$

$$Q\frac{\partial e}{\partial z} + R\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{12}w + \rho_{22}W) - b\frac{\partial}{\partial t}(w-W),$$

$$K\nabla^2 T = \rho c_v\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right). \tag{3}$$

Now we can assume the solution to the eq. (3) in the following form [13]

$$u(x,z) = C_1 e^{j\omega t - j(k_1x + k_2z)},$$

$$w(x,z) = C_2 e^{j\omega t - j(k_1x + k_2z)},$$

$$U(x,z) = C_3 e^{j\omega t - j(k_1x + k_2z)},$$

$$W(x,z) = C_4 e^{j\omega t - j(k_1x + k_2z)},$$

$$T(x,z) = C_5 e^{j\omega t - j(k_1x + k_2z)}. \tag{4}$$

In all the above C_1, C_2, C_3, C_4, C_5 are arbitrary constants, j is the complex unity and $k_i\,(i=1,2)$ is the wavenumber in the i^{th} direction such that the wavenumber $k = \sqrt{k_1^2 + k_2^2}$. Substituting eq. (4) in the eq. (3), we obtain

$$(Pk_1^2 + Nk_2^2 - \omega^2\rho_{11})C_1 + (A+N)k_1k_2C_2 + (Qk_1^2 - \omega^2\rho_{12})C_3 + Qk_1k_2C_4 - \beta jk_1C_5 + j(b\omega C_1 - b\omega C_3) = 0,$$

$$(A+N)k_1k_2C_1 + (Pk_2^2 + Nk_1^2 - \omega^2\rho_{11})C_2 + Qk_1K_2C_3 + (Qk_2^2 - \omega^2\rho_{12})C_4 - \beta jk_2C_5 + j(b\omega C_2 - b\omega C_4) = 0,$$

$$(Qk_1^2 - \omega^2\rho_{12})C_1 + Qk_1k_2C_2 + (Rk_1^2 - \omega^2\rho_{22})C_3 + Rk_1k_2C_4 + j(b\omega C_3 - b\omega C_1) = 0,$$

$$Qk_1k_2C_1 + (Qk_2^2 - \omega^2\rho_{12})C_2 + Rk_1k_2C_3 + (Rk_2^2 - \omega^2\rho_{22})C_4 + j(b\omega C_4 - b\omega C_2) = 0,$$

$$(-\beta T_0\tau_0\omega^2 k_1 / j)C_1 - (\beta T_0\tau_0\omega^2 k_2 / j)C_2 - (K(-k_1^2 - k_2^2) - \rho c_v\tau_0\omega^2)C_5 = 0. \tag{5}$$

III. NUMERICAL RESULTS

For the numerical work, the wave propagation is considered along $x-$direction. In this case $k_2 = 0$ and eq. (5) reduces to.

$$\begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0. \tag{6}$$

Due to the presence of dissipation (b) nature of the medium, waves are attenuated. For a non-trivial solution, the determinant of coefficient matrix is zero. Accordingly we obtain the complex valued frequency equation.

$$\begin{vmatrix} A_{11} & 0 \\ 0 & A_{22} \end{vmatrix} = 0. \tag{7}$$

Where $A_{ij} = |b_{ij}| + i|d_{ij}|, \quad i, j = 1, 2$ and the expression for the b_{ij} and d_{ij} are given below

$$\begin{aligned} b_{11} = {} & PRKk_1^6 - PKk_1^4\omega^2\rho_{22} + PRk_1^4\rho c_v\omega^2\tau_0 - Pk_1^2\rho c_v\tau_o\omega^4\rho_{22} - RKk_1^4\omega^2\rho_{11} - Kk_1^2\omega^4\rho_{11}\rho_{22} \\ & - Rk_1^2\rho c_v\tau_0\omega^4\rho_{11} + \rho c_v\tau_0\omega^6\rho_{11}\rho_{22} - Kk_1^2 b^2\omega^2 - \rho c_v\tau_0 b^2\omega^4 - QKk_1^6 - Q^2 k_1^4\rho c_v\tau_0\omega^2 \\ & + QKk_1^4\omega^2\rho_{12} + Qk_1^2\omega^4\rho_{12}\rho c_v\tau_0 + QKk_1^4\omega^2\rho_{12} + Qk_1^2\omega^4\rho_{12}\rho c_v\tau_0 - Kk_1^2\omega^4\rho_{12}^2 - \rho c_v\tau_0\omega^6\rho_{12} \\ & + b^2\omega^2 Kk_1^2 + b^2\omega^4\rho c_v\tau_0 + R\beta k_1^4\omega^2 T_0\tau_0 - \beta k_1^2\omega^4\rho_{22} T_0\tau_0, \end{aligned}$$

$$b_{22} = N\rho_{22}\omega^4 V_s^{-2} - Nk_1^2\omega^2\rho_{22},$$

$$\begin{aligned} d_{11} = {} & PKb\omega k_1^4 + Pk_1^2\rho c_v\tau_0 b\omega^3 - Kk_1^2\omega^3\rho_{11}b - \rho c_v\tau_0\omega^5\rho_{11}b + RKk_1^4 b\omega - Kk_1^2 b\omega^3\rho_{22} + Rk_1^2\rho c_v\tau_0 b\omega^3 \\ & - \rho c_v\tau_0 b\omega^5\rho_{12} + QKk_1^4 b\omega + Qk_1^2 b\omega^3\rho c_v\tau_0 - Kk_1^2 b\omega^3\rho_{12} - \rho c_v\tau_0 b\omega^5\rho_{12} + QKk_1^4 b\omega \\ & + Qk_1^2 b\omega^3\rho c_v\tau_0 + Kk_1^2\omega^2\rho_{12}b\omega - \rho c_v\tau_0\omega^5\rho_{12}b\omega + \beta k_1^2 b\omega^3 T_0\tau_0, \end{aligned}$$

$$d_{12} = Nk_1^2 b\omega - b\omega^3\rho_{11} - b\omega^3\rho_{22} - 2\omega^3\rho_{12}b,$$

and $V_s^2 = \dfrac{N\rho_{22}}{\rho_{11}\rho_{22} - \rho_{12}^2}$ is the shear wave velocity.

a.Particular case

In the absence of dissipation coefficient $(b = 0)$ frequency equation (7) reduces to the following form

$$\begin{vmatrix} C_{11} & 0 \\ 0 & C_{22} \end{vmatrix} = 0. \quad (8)$$

Where

$$C_{11} = PRKk_1^6 + PR\rho c_v k_1^4 \omega^2 \tau_0 - PKk_1^4 \omega^2 \rho_{22} - P\rho c_v k_1^2 \omega^4 \rho_{22}\tau_0 - RKk_1^4\ \omega^2 \rho_{11} - R\rho c_v \omega^4 k_1^2 \rho_{11}\tau_0$$
$$+ Kk_1^2\omega^4\rho_{11}\rho_{22} + \rho c_v \omega^6 \tau_0 \rho_{11}\rho_{22} - Q^2 Kk_1^6 + 2QKk_1^4\omega^2\rho_{12} - Kk_1^2\omega^4\rho_{12}^2 - Q^2\rho c_v \omega^2 k_1^4 \tau_0$$
$$+ 2Q\rho c_v k_1^2 \omega^4 \tau_0 \rho_{12} - \rho c_v \omega^6 \rho_{12}\tau_0 + \beta^2 T_0 R k_1^4 \omega^2 \tau_0 - \beta^2 T_0 k_1^2 \omega^4 \tau_0 \rho_{22},$$

$$C_{22} = Nk_1^2 - N\frac{\omega^2}{V_S^2}.$$

The frequency equation (7) and (8) is investigated for the following poroelastic solids [14, 15, 16]. The thermoelastic constant values are given in [17].

For sandstone saturated with kerosene (material-1)

$$A = 0.4436\times 10^{10}\,N/m^2,\quad N = 0.2765\times 10^{10}\,N/m^2,\quad Q = 0.07635\times 10^{10}\,N/m^2,$$
$$R = 0.0326\times 10^{10}\,N/m^2,\quad \rho_{11} = 1.926137\times 10^3\,kg/m^3,\quad \rho_{12} = -0.002137\times 10^3\,kg/m^3,$$
$$\rho_{22} = 0.21537\times 10^3\,kg/m^3,\quad \beta = 3.2\times 10^{-4}\,1/^0k,\quad K = 0.13\omega/m^0k,\quad \rho c_v = 1.67\times 10^6\,J/m^3k. \quad (9)$$

For sandstone saturated with water (material-2)

$$A = 0.306\times 10^{10}\,N/m^2,\quad N = 0.922\times 10^{10}\,N/m^2,\quad Q = 0.013\times 10^{10}\,N/m^2,$$
$$R = 0.0637\times 10^{10}\,N/m^2,\quad \rho_{11} = 1.90302\times 10^3\,kg/m^3,\quad \rho_{12} = 0, \rho_{22} = 0.2268\times 10^3\,kg/m^3,$$
$$\beta = 6.6\times 10^{-5}\,1/^0k,\quad K = 0.607\omega/m^0k,\quad \rho c_v = 4.17\times 10^6\,J/m^3k. \quad (10)$$

For Berea sandstone (material-3)

$$A = 8224\times 10^6\,N/m^2,\quad N = 70199N/m^2,\quad Q = 98199N/m^2, R = 38\times 10^7\,N/m^2,$$
$$\rho_{11} = 2415.2kg/m^3,\quad \rho_{12} = -300kg/m^3, \rho_{22} = 500kg/m^3,\quad \beta = 1.5\times 10^{-6}\,1/^0k,$$
$$K = 2.34\omega/m^0k,\quad \rho c_v = 1.76\times 10^6\,J/m^3k,\quad T_0 = 296^0k,\quad \tau_0 = 10^{-3}s \quad (11)$$

The complex frequency (7) gives frequency and attenuation coefficient as a function of wavenumber. The real part of eq. (7) gives frequency of wave, whereas Q^{-1}= (2imaginary part of eq. (7)) / (real part of eq. (7)) gives attenuation coefficient. Substituting the values of eq. (9), eq. (10) and (11) in the frequency equations (7) frequency, attenuation coefficient are computed as a function of wavenumber. Frequency and attenuation are computed using the bisection method implemented in MATLAB, and results are depicted in figure 1-5 graphically. Figure 1&2

shows the plots of frequency against wavenumber in the presence of dissipation $(b = 0.01)$ and $(b = 0.1)$. From figures 1, 2 it is observed that the frequency of material-1, 2 are almost steady and constant and it is also clear that frequency of material-1, 2 are greater than material-3. Figure 3&4 shows the plots of attenuation against wavenumber in the presence of dissipation $(b = 0.01)$ and $(b = 0.1)$. From figures 3, 4 it is observed that attenuation of material-3 is greater than that of material-1 and 2. From figures 1-4, it is also clear that as the dissipation increases attenuation and frequency increases. As the dissipation increases the values of frequency and attenuation are almost constant for $(b = 0.1)$ and $(b = 1)$. Figure 5 shows the plots of frequency against wavenumber in the absence of dissipation. It is observed that as the wavenumber increases frequency increases for all the three materials. From figures 1-5 it is also clear that frequency is greater in the absence of dissipation than that of frequency and attenuation in the presence of dissipation.

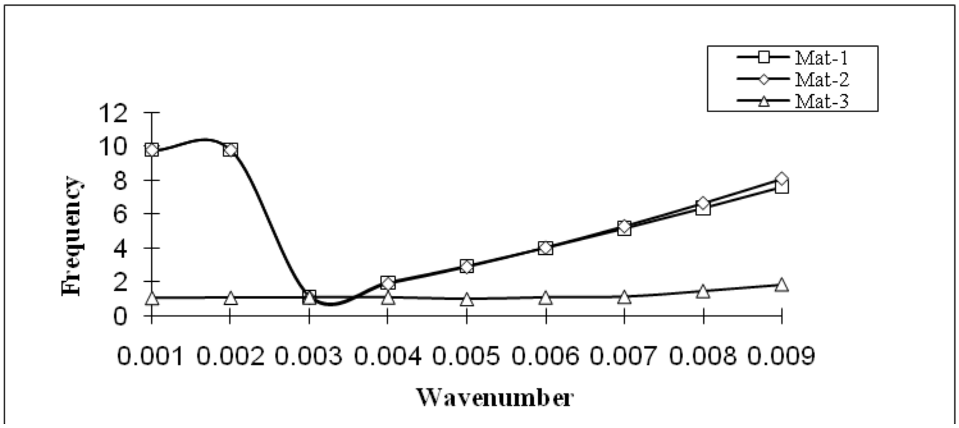

Fig: 1 Variation of frequency with wavenumber at dissipation (b=0.01)

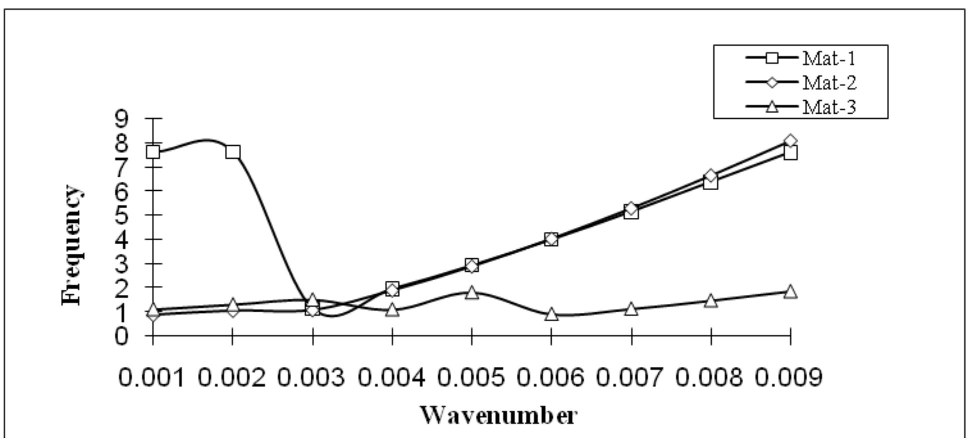

Fig: 2 Variation of frequency with wavenumber at dissipation (b=0.1)

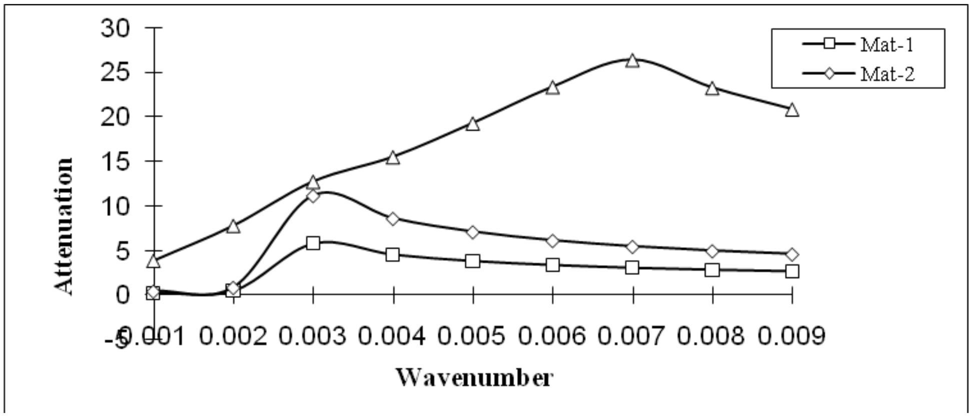

Fig: 3 Variation of attenuation with the wavenumber at dissipation (b=0.01)

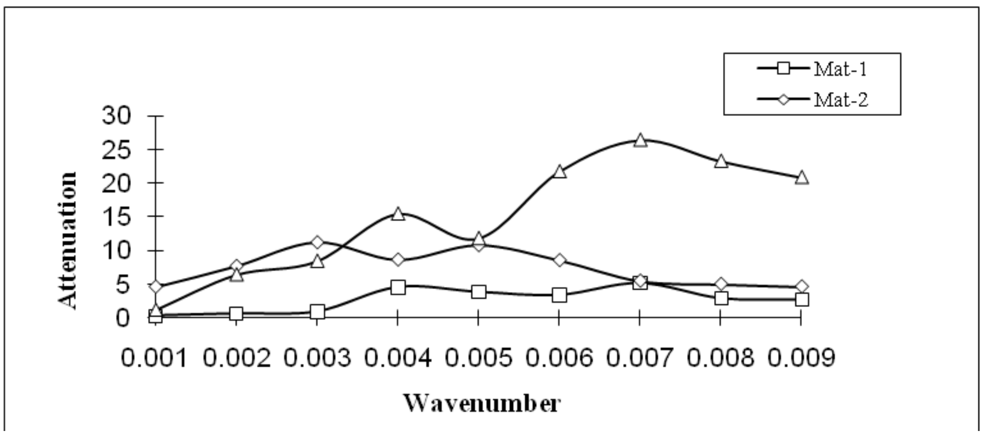

Fig: 4 Variation of attenuation with wavenumber at dissipation (b=0.1)

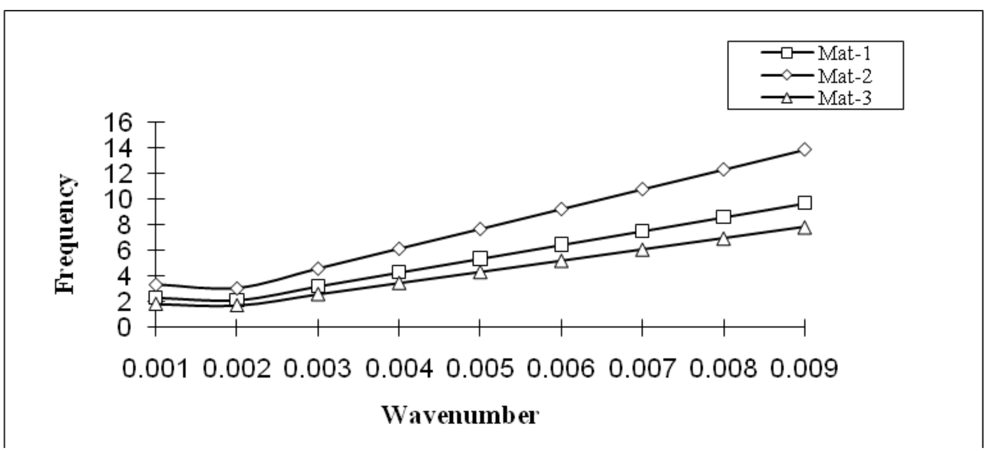

Fig: 5 Variation of frequency with wavenumber at dissipation (b=0)

IV. CONCLUSION

Employing Biot's theory, plane strain vibrations in thermoporoelastic plates are investigated. Pertinent constitutive relations and governing equations are derived. Frequency is computed for three poroelastic solids. The complex valued frequency equation is reduced to real valued equations which gives that frequency and attenuation. Attenuation values are greater than that of frequency. In the absence of dissipation as the wavenumber increases frequency increases for all the three materials.Similar analysis can be made for different poroelastic constants if pertinent values are available.

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