

# ANALYTICAL STUDY OF PID AND LQR: APPLICATION TO ROBOTIC MANIPULATOR

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**Abstract:** PID and LQR are the basic controllers but industries are using PID as the conventional controller for many years. In these paper we have performed the analytical study of the two controlling strategies for controlling the robotic manipulator. A classical Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) have been simulated in NI Lab View for this and we have considered PMDC motor for controlling the robotic manipulator for this. This strategy used basic comparison of time domain and frequency domain parameters of the PID and LQR simulation. A real time embedded evaluation board is used for this.

**Keywords:** - PID, LQR, LabView, MyRio, G- programming

## I INTRODUCTION

Our aim is to compare the control strategy between Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) controller while controlling a DC motor equipped in a robotic arm. Before implementation of this project we have visited several automation industries in order to observe the PID applications and its controlling strategy. Similarly we have visited sites where LQR controlling strategy is being used. Conventionally many industries use PID controller for the controlling some examples are temperature control in AC, cruise control in vehicles, pressure controlling. Let's take example of using PID in cruise control, in cruise control the user i.e. drivers sets the desired speed as the output which is compared with the reference point for the controller, so after setting this desired speed the vehicle tracking is done on how the vehicles attends the desired speed in what time etc. such parameters are being calculated. PID is also used in Air Conditioners so as to attend a desired temperature and to maintain it once the desired temperature is achieved without fluctuations then steady state is achieved. The controller which achieves steady state in less time with

less fluctuations is said to have more stability and more efficiency. The simulation of PID in MATLAB is also possible but then we have to use DAC to interface the DC motor with the simulation but since MyRio is more effective as compared to DAC so we have selected NI's LabView and MyRio to control the speed of DC motor. There are some alternatives available in the market for PID and LQR they are the MPC controller and fuzzy logic controller we Identify applicable funding agency here. If none, delete this. have analyzed them as well with their proper applications. But we have concluded that in order to control the DC motor PID and LQR provides best results. While working with LQR the simulation of the LQR with G-programming was a challenging task since LQR using optimal control strategy unlike the PID. For designing the LQR in G-programming first of all the cost function is taken into consideration and the its being simulated. After surveying we have observed that in case of cruise control in vehicles the PID often take more to reach the steady state as compared to LQR this my be because of the optimal gain strategy which is used in used in LQR. Similar observations have been made in the case if air conditioners, in controlling the temperature of air conditioner some time PID controller shows more fluctuations. These are some of the observations of our survey in automation sector.

## II ROBOTIC MANIPULATOR

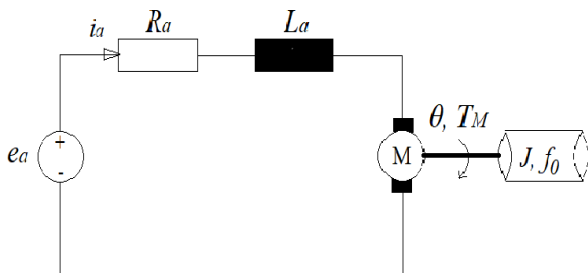
### A. Selection Criteria of Motor

While operating the robotic manipulator we have to use a motor and according to the selection criteria of the motor we have to select proper motor. While considering the robotics manipulator we had two alternatives either PMDC motor or stepper motor. Comparing stepper motor with PMDC motor we know that stepper motor has more precision and accuracy than the PMDC motor, but controlling of PMDC motor would be an easy task we have selected PMDC motor to

be equipped with robotic manipulator. The robotic manipulator which we have been controlling is of one Degree of Freedom (DOF) and it has an end effector equipped with it for various tasks and applications.

**B. Mathematical Analysis of Motor**

Mathematical modelling of permanent magnet DC motor is presented in this section. Torque and electrical equations described and have been considered for derivation of the model. The electrical circuit of the motor is shown in Fig. 1.



**Fig. 1.**

It can be represented by a voltage source ( $e_a$ ) across the coil of the armature. The electrical equivalent of armature coil can be denoted by an inductance ( $L_a$ ) in series with a resistance ( $R_a$ ) in series with an induced voltage or back electromotive force (emf) ( $e_b$ ) which opposes the voltage source. Rotation of electrical coil through fixed flux lines of permanent magnets generates back emf. A differential equations for the electrical circuit shown in Fig. 1 can be derived by applying Kirchoff's voltage law. Using Ohm's and Kirchoff's law the sum of all voltages around a loop is given by,

$$L \frac{di_a}{dt} + R i_a + e_b = e_a \tag{1}$$

Where  $i_a$  is the armature current and  $di_a/dt$  is the change of current through coil with respect to time. The back emf is obtained by,

$$e_b = K_m \omega_m \tag{2}$$

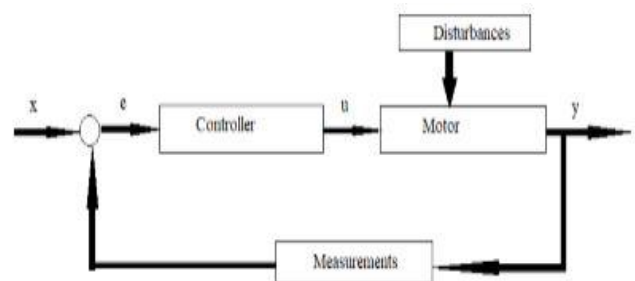
$K_m$  being the back emf constant which is related with flux density of permanent magnets, reluctance of the iron core of armature, and the number of turns of the armature winding and  $m$  is the rotational speed of rotor. Motor exerts a torque due to the supplied voltages at stator and rotor. The mechanical structure characterized by the rotor inertia  $J$  and the viscous friction coefficient  $B_m$  is acted upon by this torque. It has also to be taken

into account that in any operating environment a load torque is exerted on the motor; then, if  $T_l$  is load torque. The electrical and mechanical parameters have been calculated for the PMDC motor by practically measuring all elements of the motor.

Notation	Description	Value
$K_m$	Motor torque constant	0.0192
$J$	Moment of inertia	0.000163 2
$B_m$	Viscous friction constant	0.000894 73
$R_a$	Armature resistance	2.8
$L_a$	Armature inductance	1.2258

**III CONTROLLING STRATEGY**

In this section we will discuss closed-loop control of DC motor and design of PID, LQR and MPC controllers. Figure shows the closed-loop system of DC motor control. In the Figure,  $r$  is the desired reference,  $u$  is the obtained input voltage from any controller,  $y$  is the measured output that is speed of the motor, and  $e$  is the error between reference and output. Disturbance acting on the motor speed is also shown in Fig. 2.



**Fig. 2.**

**A. Proportional Integral Derivative Controller**

The name Proportional-Integral-Derivative (PID) controller itself suggests that the algorithm consists of three basic modes, the Proportional, the Integral and the Derivative mode. The proportional action adjusts the output signal in direct proportion with controller input (here error signal). The adjustable parameter here is controller gain  $K_p$ . A proportional controller reduces the error but doesn't eliminate it i.e. an offset between actual and ideal will always exist. To eliminate this offset integral mode is used. The adjustable parameter here is  $T_I$  of the controller also known as integral time. The last mode is the derivative

mode which anticipates by looking at the time rate of change of controlled variable.  $TD$  is the rate time which can be adjusted to adjust derivative action. PID in continuous time form can be stated as:

$$u_t = K_p(e_t + \int e_t dt + T_d \frac{de_t}{dt}) \quad (3)$$

### B. Linear Quadratic Regulator

Linear Quadratic Regulator (LQR) design technique is well known in modern optimal control theory and has been widely used in many applications. Optimal control problem has characteristics like minimizing scalar function by adjustment of manipulated variable and constraints which can be need to be essentially satisfied continuously or satisfied at the end of optimization problem. Also it is characterized by the horizon which theoretically is infinite but in practice is finite. Consider the discrete time state space model of the system,

$$Xk = AX + BU \quad (4)$$

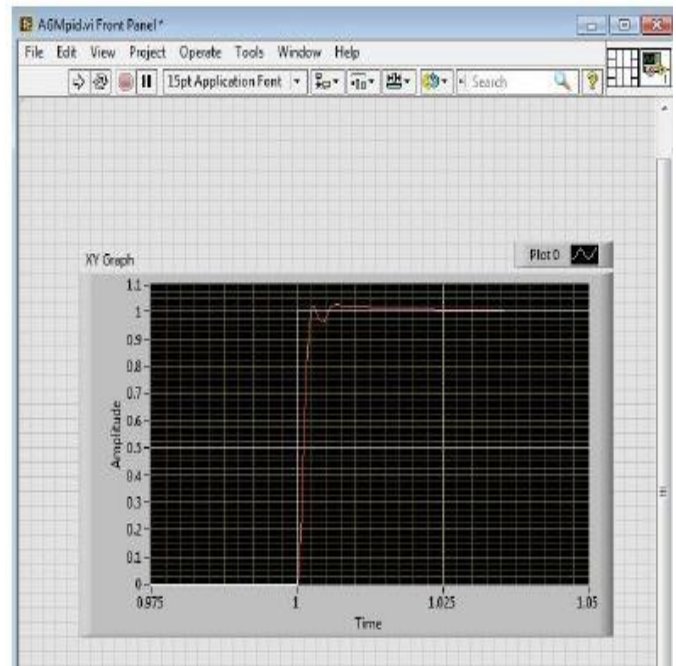
The discrete LQR problem is to search for the  $N$  future optimal control moves to minimize the cost function.

$$J = \frac{1}{2} X^N n S_f X_n + \frac{1}{2} \sum (X^T k Q_k Xk + U^T k R_k U_k) \quad (5)$$

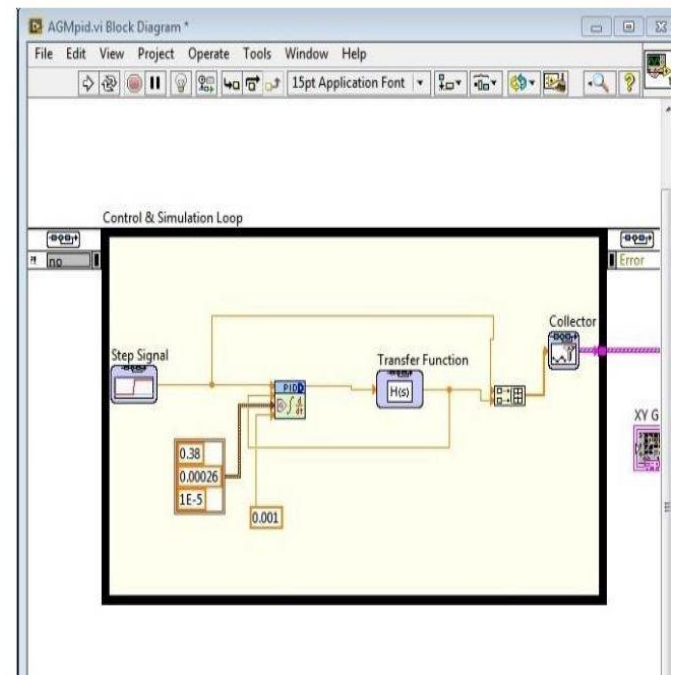
where  $Qk$  and  $Sf$  are positive semi definite while  $Rk$  is a positive definite matrix. The LQR algorithm is essentially an automated way of finding an appropriate state feedback controller. As such, it is not uncommon for control engineers to prefer alternative methods, like full state feedback, also known as pole placement, in which there is a clearer relationship between controller parameters and controller behavior. Difficulty in finding the right weighting factors limits the application of the LQR based controller synthesis.

### IV RESULTS

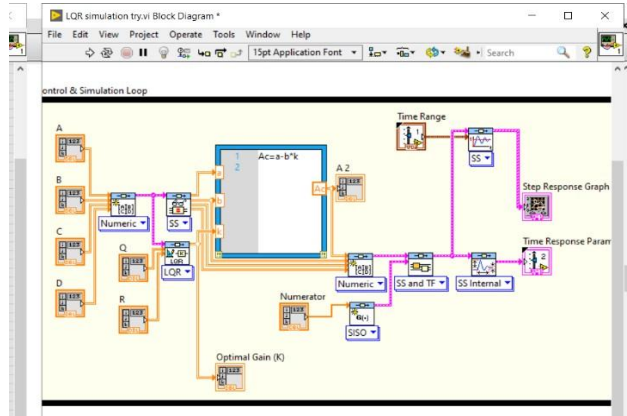
We have performed the G programming in NI LabView for designing and implementation of the PID and LQR controller. The LQR algorithm is essentially an automated way of finding an appropriate state feedback controller. As such, it is not uncommon for control engineers to prefer alternative methods, like full state feedback, also known as pole placement, in which there is a clearer relationship between controller parameters and controller behavior. Difficulty in finding the right weighting factors limits the application of the LQR based controller synthesis.



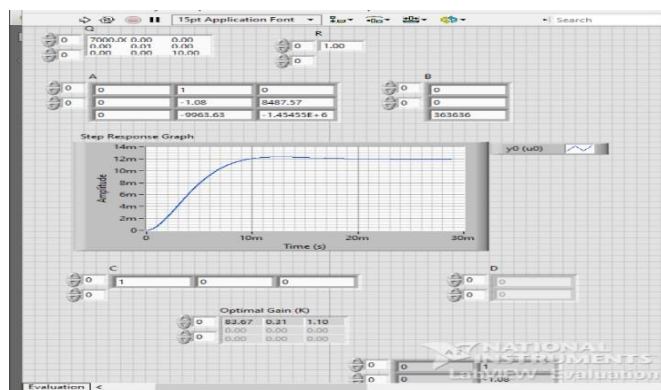
The actual designing of PID is done in the block diagram panel window of the LabView. For which the step signal is given as the reference input the PID block and parameters are also provided in it. This whole set up is under the control and simulation loop while a graph block is connected to it. The actual implementation of the PID controller in block diagram is shown below,



The design of LQR simulation is given below. The simulation is performed in the LabView software by using Block diagram panel window.



The output obtained from the above simulation is given in the below diagram.



## V.CONCLUSION

### A. Analysis of Result

After performing the simulation and the hardware of the project we have achieved some results. These results also have been observed in both forms software results as well as the hardware results. Now let's take a look at the results of the project. Now take a close look at both the lines PID as well as well LQR so have observed the following points while considering the time domain analysis. In the above results we have seen the following observations,

- **Peak Overshoot-** PID has more peak over shoot as compared to the LQR, like here set point is 1 but PID reaches to the value of 1.8 which is much more than the set point.
- **Settling Time-** Since PID has greater value pf the peak overshoot the settling time for PID is more as compared to the settling time of the LQR.
- **Rise time** - The rise time is less for the LQR as compared to the rise time of the PID.

## ACKNOWLEDGMENT

Many helpful hands were there in our research. This work would have not been complete without my group mates and Dr. Prashant Thackrey (Dean of instrumentation and control department, Amravati University) who has provided his special guidance to us. I would like to thank our guide Prashant Aher for his enormous support and understanding, He has helped us in all possible ways with his great expertise and experience in the field of control system and robotics.

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