

# Study of MHD Hybrid Nanofluid Flow over a Non-linear Stretched Sheet

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**Abstract:** In this paper, the magnetohydrodynamic (MHD) hybrid nanofluid flow over a non-linear stretched sheet is mathematically designed and analysed. The hybrid nanofluid is composed of two nanoparticles suspended in a base fluid, and the effect of a magnetic field is considered for the flow characteristics. The governing equations for mass, momentum, and energy transport are derived, and a similarity transformation is applied to convert these equations into a system of ordinary differential equations. The numerical solution of these equations is obtained using the RKF method, and the results are analysed to study the impact of various parameters, such as the magnetic parameter and nanoparticle volume fraction on the flow and heat transfer behaviour. The results indicate that the inclusion of hybrid nanofluids enhances the heat transfer rate and affects the velocity distribution significantly.

**Keywords:** *Non-linear Stretching, MHD, Hybrid Nanofluid*

## I. INTRODUCTION:

Several engineering processes rely on the boundary layer flow, which occurs across a continuous solid surface that is in motion. Extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growth, and countless more processes all involve heat-treated materials traveling between a feed roll and a wind-up roll. The rate of cooling and the procedure of stretching determine the final product's intended features in these circumstances. The ground-breaking research of Crane [1] is the source of the dynamics of the fluid flow in the boundary layer across a stretching surface. Recent years have seen a rise in interest in MHD flow because of its many industrial applications and adjustable heat transfer rate. The effect of MHD on the transfer of heat and flow over stretching surfaces was investigated by Hayat et al. [2].

Many industrial processes rely on nonlinear stretching sheets, such as the cooling bath used in the condensation process of metal plates and the extrusion of polymer sheets using a dye, among many others. The process of making a polymer sheet involves spreading melted material through a slit, which then stretches to achieve the desired width of the sheet. The properties of the finished product are affected by the stretching rate of the raw material that was utilized to make the sheet. In view of this, fluid's flow across a nonlinear stretching sheet were investigated by Vajravelu [3]. Cortell [4] studied the viscous flow and heat transfer over a nonlinearly stretching sheet.

Makkar and Poply [5] investigated the effect of external velocities on the transport of Casson nanofluids' flow, heat, and mass through a non-linear stretching sheet. The numerical aspects of MHD casson nanofluid flow towards a non-linear stretching sheet has been studied in [6]. A nonlinear stretching sheet has been the subject of research into radiative hybrid nanofluid thermal flow [7]. Waini et al. [8] examined the flow of hybrid nanofluids and heat transfer across a nonlinear permeable stretching/shrinking

surface. Heat and mass transport by bioconvection through a nonlinear stretching sheet involving hybrid nanofluids, joule dissipation, and entropy formation was examined in [9]. Numerical analysis of hybrid nanofluid flow over a nonlinear stretching sheet with viscous dissipation, joule heating effects has been discussed in [10].

## II. Mathematical Formulation

Consider a steady, incompressible, magnetohydrodynamic flow of a hybrid nanofluid over a non-linear stretched sheet. The base fluid is a water-based nanofluid that contains two types of nanoparticles: copper (Cu) and alumina (Al<sub>2</sub>O<sub>3</sub>). The magnetic field is applied perpendicular to the flow direction. The schematic diagram is represented in Figure 1.

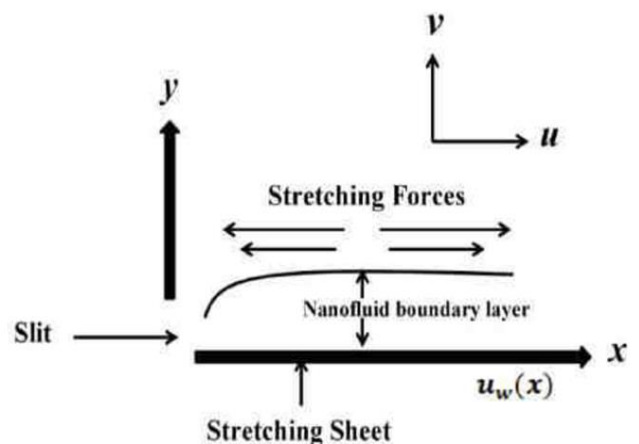


Figure 1. Schematic diagram

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B^2 u$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}} \frac{\partial^2 T}{\partial y^2}$$

In these equations, the effects of nanoparticles are incorporated by modifying the fluid properties, including the density, dynamic viscosity, thermal conductivity, and specific heat capacity.

Boundary conditions are

$$\left. \begin{aligned} u = u_w = ax^n, \quad v = 0, \quad T = T_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty \end{aligned} \right\}$$

Similarity Variables

$$\eta = \sqrt{\frac{a(n+1)}{2\nu}} y x^{\frac{n-1}{2}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$u = ax^n f'(\eta), \quad v = -\sqrt{\frac{av(n+1)}{2}} x^{\frac{n-1}{2}} \left( f(\eta) + \left(\frac{n-1}{n+1}\right) \eta f'(\eta) \right)$$

Reduced ordinary differential equations are

$$\frac{\mu_{hnf}}{\mu_f} f'''' + \frac{\rho_{hnf}}{\rho_f} \left( f f'' - \frac{2n}{n+1} f'^2 \right) - \frac{\sigma_{hnf}}{\sigma_f} \frac{2}{n+1} M f' = 0$$

$$\frac{k_{hnf}}{k_f} \theta'' + \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} Pr f \theta' = 0$$

Corresponding Boundary Conditions

$$\left. \begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \\ f'(\infty) = 0, \quad \theta(\infty) = 0 \end{aligned} \right\}$$

### III. Results and Discussion:

By employing the RKF-45 technique, the dimensionless ordinary differential equations (5)–(6) and boundary conditions (7) are both resolved. Among the many noteworthy outcomes of the research on MHD hybrid nanofluid flow over a stretched linear sheet are:

Figure 2 shows the effect of a magnetic field on the velocity profile of the hybrid nanofluid. The opposing force to the fluid flow, the Lorentz force, grows in direct proportion to the amount of M. Consequently, the momentum is decreasing as M increases due to a stronger Lorentz force, which in turn causes a higher resistance to the fluid flow. Figure 3 shows the velocity profiles for different values of n. Velocity increases with an increase in nonlinearity.

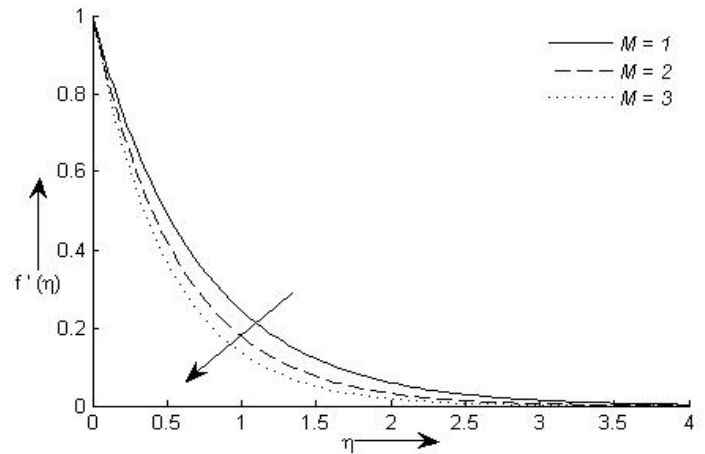


Figure 2. Velocity profiles for M

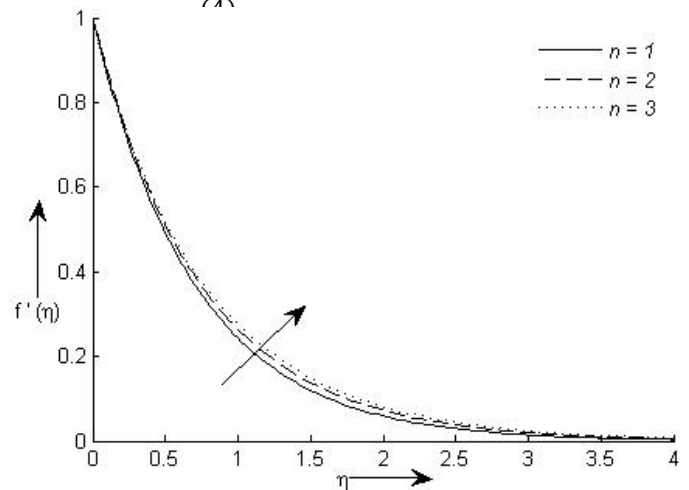


Figure 3. Velocity profiles for n

Figure 4 shows the effect of a magnetic field on the temperature profile. The reduced flow velocity is accompanied by an increase in temperature due to the increased heat conduction capabilities of the nanoparticles. Figure 5 shows the effect on temperature with distinct n. Temperature decreases with an increase in n.

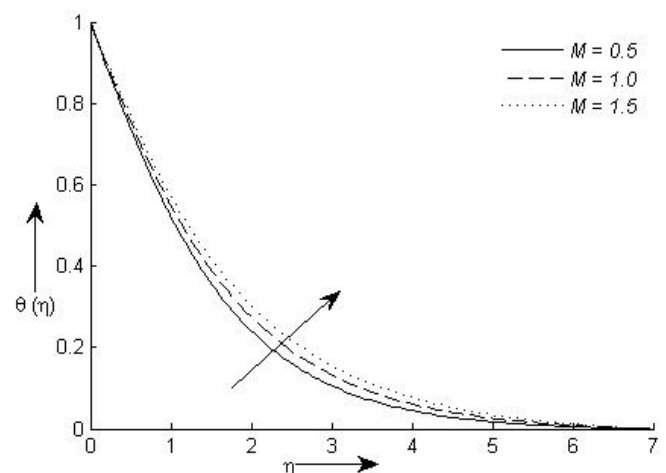


Figure 4. Temperature profiles for M

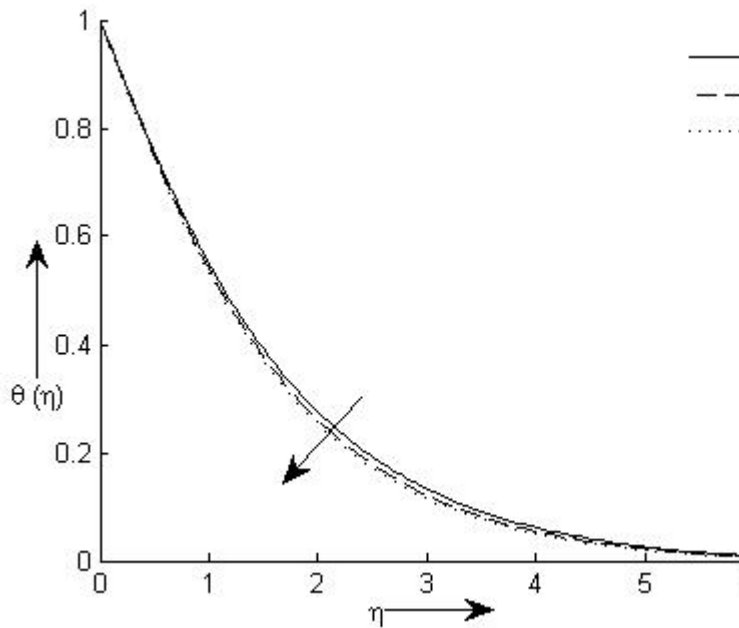


Figure 5. Temperature profiles for  $n$

The hybrid nanofluid expressions improved heat transfer characteristics compared to conventional fluids. This is because the two types of nanoparticles provide enhanced thermal conductivity, leading to better heat dissipation from the surface. The temperature gradient is higher for hybrid nanofluids, indicating faster heat transfer from the surface to the surrounding fluid.

#### IV. Conclusions

This research emphasizes the impacts on fluid flow and heat transfer caused by MHD, hybrid nanofluid composition, and non-linear sheet stretching. Using hybrid nanofluids improves heat transfer efficiency and thermal conductivity, and a magnetic field has a substantial impact on the velocity and temperature profiles. An additional important factor in the flow properties and heat transfer rate is the sheet's non-linear stretching behaviour. The findings shed light on how to improve heat transfer in engineering settings where stretched sheets are used, for example, to cool metal surfaces and extrude polymers.

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