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ESTIMATION OF LOCATION (θ) AND SCALE (λ) OF TWO-PARAMETER HALF LOGISTIC -RAYLEIGH DISTRIBUTION (HLRD) USING MEDIAN RANKS REGRESSION METHOD

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Abstract: - In this paper, we propose the estimation of Location (θ) and Scale (λ) parameters using the Median Rank method (Benard's approximation) . We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under complete sample based on 1000 simulations to assess the performance of the estimators.

Keywords: *Two parameter HLRD, Median Rank method (Benard's approximation),Montecarlo Simulation.*

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I INTRODUCTION

Generally in many of the situations, we face some type of situations of non monotonic failure rates to supervise the reliability analysis of the data. In order to model such data, proposed by Aarset *et al* (1987) and Venkataraman *et al* (1988) proposed Least squares estimators and Weighted Least squares estimators of a Beta distribution. Madholkar *et al* (1995) present an extension of the Weibull family that not only contains unimodel distribution with bath tub failure rates but also allows for a broader class of monotone hazard rates and is computationally convenient for censored data. They named their extended version as “Exponentiated Weibull Family”. On similar lines Gupta and Kundu (2001b) proposed a new model called generalized exponential distribution. A generalized (type – II) version of logistic distribution was considered and some interesting properties of the distribution were derived by Balakrishnan and Hassain (2007). Ramakrishna (2008) studied the Type I generalized half logistic distribution scale (σ) and shape (θ) parameters estimation using the least square method in two step estimation methods. Torabi and Bagheri (2010) considered different parameter estimation methods in extended generalized half logistic distribution for censored as well as complete sample. Rama Mohan and Anjaneyulu (2011) studied how the least square method be good for estimating the parameters in two parameter Weibull distribution from an optimally constructed grouped sample. In this paper, in Section - 2, we develop a practical approach for estimating the location(θ) and scale (λ) parameters of the HLRD using the median rank regression method. We therefore employ these estimates as the median ranking method.

In Section – 3 we present the observations and the conclusions are based on the simulation results with the numerical example.

Let x_1, x_2,..., x_n be a random sample of size n from HLRD (θ,λ), its Probability Density Function (PDF), cumulative distribution function(CDF) and Hazard Function (HF) are given by

$$f(x;\theta,\lambda)=\frac{4\lambda(x-\theta)e^{-\lambda(x-\theta)^2}}{\left[1+e^{-\lambda(x-\theta)^2}\right]^2};x>\theta,\lambda>0 \quad ...(1.1)$$

$$F(x;\theta,\lambda)=\frac{1-e^{-\lambda(x-\theta)^2}}{1+e^{-\lambda(x-\theta)^2}};x>\theta,\lambda>0 \quad ...(1.2)$$

$$h(x)=\frac{2\lambda(x-\theta)}{\left[1+e^{-\lambda(x-\theta)^2}\right]} \quad ...(1.3)$$

II. ESTIMATION OF LOCATION (Θ) AND SCALE(λ) PARAMETERS OF TWO PARAMETER HALF LOGISTIC – Rayleigh distribution

A. HLRD using Median Ranks Method

Let $x_{(1)} < x_{(2)} < x_{(3)} \ldots\ldots\ldots < x_{(n)}$ be an ordered sample of size 'n' from Half Logistic Rayleigh Distribution with the parameters Location (θ) and Scale(λ). Then the CDF is given as in equation (1.2), can be written as

$$1+F(x)=\frac{2}{1+e^{-\lambda(x-\theta)^2}} \quad ..(2.1)$$

and

$$1-F(x)=\frac{2e^{-\lambda(x-\theta)^2}}{1+e^{-\lambda(x-\theta)^2}} \quad ...(2.2)$$

$$\frac{1+F(x)}{1-F(x)}=e^{\lambda(x-\theta)^2} \quad ...(2.3)$$

Taking Logarithm on both side of equation (2.3), we get

$$x=\theta+\frac{1}{\sqrt{\lambda}}[\log Y]^{1/2} \quad ..(2.4)$$

Where $Y=\left(\frac{1+F(X)}{1-F(X)}\right)$

From the least square parameter estimation method (also known as regression analysis), let us consider

$$A=\theta,\ \hat{\theta}=\hat{A},\ B=\frac{1}{\sqrt{\lambda}},\ \hat{\lambda}=\frac{1}{\sqrt{B}}. \quad ...(2.5)$$

and $U=\log\left(\frac{1+F(x)}{1-F(x)}\right)^{1/2},\ V=x$

F (x) is obtained by using the median rank method (also known as Benard's approximation), which is a good estimate of the median rank estimate. Benard's median rank was used because it showed the best performance and was the most used rank for prediction (F (x)). The procedure for ranking the complete data is as follows:

1. List the time to failure data from small to large.
2. Use Benard's formula to assign median ranks to each failure.
3. Estimate the θ and λ by Equations (2.6) and (2.7).
4. $F(x)=\frac{i-0.3}{N+0.4}$, i= 1, 2, ..., N.

Where i → The ranked position of data point and

N → The total number of units in the sample

$$\hat{A}=\frac{\left(\sum_{i=1}^{n}v_i^2\right)\left(\sum_{i=1}^{n}u_i\right)-\left(\sum_{i=1}^{n}v_i\right)\left(\sum_{i=1}^{n}v_iu_i\right)}{\left(n\sum_{i=1}^{n}v_i^2\right)-\left(\sum_{i=1}^{n}v_i\right)^2} \quad ...(2.6)$$

B. Simulation study:

In order to obtain the median ranks method estimators of Location (θ)and Scale (λ) is used to obtain estimators and to study their predictive properties by Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\hat{\xi}_{lm}$ is Median Ranks Method estimate of $\hat{\xi}_m$, m=1, 2 where ξ_m is a general notation that can be replaced by $\xi_1=\lambda, \xi_2=\theta$ based on sample l, (l=1,2,…,r), then the Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

$$\text{Average Estimate }(\hat{\psi}_m)=\frac{\sum_{i=1}^{r}\hat{\psi}_{lm}}{r}$$

$$\text{Variance}(\hat{\psi}_m)=\frac{\sum_{i=1}^{r}(\hat{\psi}_{lm}-\overline{\hat{\psi}_{lm}})^2}{r}$$

$$\text{Mean Absolute Deviation}=\frac{\sum_{i=1}^{r}Med\left(\left|\hat{\psi}_{lm}-\overline{\hat{\psi}_{lm}}\right|\right)}{r}$$

$$\text{Mean Square Error }(\hat{\psi}_m)=\frac{\sum_{i=1}^{r}(\hat{\psi}_{lm}-\psi_m)^2}{r}$$

$$\text{Relative Absolute Bias}(\hat{\psi}_m)=\frac{\sum_{i=1}^{r}|(\hat{\psi}_{lm}-\psi_m)|}{r\psi_m}$$

$$\text{Relative Error}(\hat{\psi}_m)=\frac{1}{r}\left(\frac{\sum_{i=1}^{r}MSE\sqrt{(\hat{\psi}_{lm})}}{\psi_m}\right)^2$$

III. OBSERVATIONS FROM SIMULATION RESULTS AND CONCLUSIONS

1. The Average Estimate (AE), Variance (VAR), Standard deviation (STD), MeanSquare Error (MSE) and Relative Absolute Bias (RAB) are independent of true values of the parameters of Location (θ) and Scale (λ) by observing simulated data sets .
2. Average Estimate (AE) of Location parameter ($\hat{\theta}$) and Scale parameter ($\hat{\lambda}$)
3. in Median Ranks Method are decreasing when sample size (N) is increasing.

4. Variance (VAR) of Location parameter($\hat{\theta}$) and Scale parameter($\hat{\lambda}$) in Media Ranks Method are increasing when sample size (N) is increasing.
5. Mean Absolute Deviation (MAD) of Location parameter ($\hat{\theta}$) and Scale parameter ($\hat{\lambda}$) in Median Ranks Method and decreasing when sample size (N) is increasing.
6. Mean Square Error (MSE) of Location parameter ($\hat{\theta}$) and Scale parameter ($\hat{\lambda}$) in Median Ranks Method are increasing when sample size (N) is increasing.
7. Relative Absolute Bias (RAB) of Location parameter ($\hat{\theta}$) and Scale parameter ($\hat{\lambda}$) by Median Ranks Method were increasing when sample size (N) is increasing

Simulated data set:

We evaluate the performance of the Maximum Likelihood method for estimating the HLRD (θ,λ), Newton-Raphson simulation for a two parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=50(50)500 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the scale and Location parameters are unknown population parameters of HLRD. Population parameters Location=2.5 and Scale =3 in Table 2.1.

Table 2.1

Maximum Likelihood method –HLRD

Sample size	Para meters	Average Estimation	Variance	MAD	MSE	RAB	RE
50	$\hat{\theta}$	2.166777	0.209407	0.429869	0.6942609	0.1388705	0.4166116
	$\hat{\lambda}$	3.701129	0.050867	0.296849	0.4427107	0.2402258	0.6005645
100	$\hat{\theta}$	2.336365	0.272524	0.396153	0.4404112	0.2212116	0.3318174
	$\hat{\lambda}$	3.741033	0.059552	0.35041	0.5401623	0.4964131	0.6205164
150	$\hat{\theta}$	3.05797	0.477578	0.860012	0.0033606	0.0289852	0.0289852
	$\hat{\lambda}$	3.57155	0.21574	0.664126	0.1482192	0.64293	0.535775
200	$\hat{\theta}$	2.965945	0.733181	0.107373	0.0011598	0.0227035	0.0170276
	$\hat{\lambda}$	2.812918	0.573344	0.97553	0.0979178	0.2503346	0.1564591
250	$\hat{\theta}$	3.256239	0.222201	0.61589	0.0656584	0.2135325	0.1281195
	$\hat{\lambda}$	2.760442	0.648139	0.097603	0.0678298	0.2604416	0.1302208
300	$\hat{\theta}$	3.277651	0.22294	0.602135	0.0770898	0.2776506	0.1388253
	$\hat{\lambda}$	2.779467	0.561454	0.970972	0.0781017	0.3353602	0.1397334
350	$\hat{\theta}$	3.269612	0.180221	0.48347	0.0726908	0.3145478	0.1348062
	$\hat{\lambda}$	2.881794	0.525727	0.900295	0.1457668	0.5345118	0.1908971
400	$\hat{\theta}$	3.501849	0.107994	0.408219	0.2518524	0.669132	0.2509245
	$\hat{\lambda}$	3.024585	0.37652	0.810519	0.2751891	0.8393355	0.2622924
450	$\hat{\theta}$	3.05651	0.347369	0.780057	0.0031934	0.084765	0.028255
	$\hat{\lambda}$	2.535092	0.731712	0.105705	0.0012315	0.01	0.0175461
500	$\hat{\theta}$	3.041695	0.366581	0.807155	0.0017385	0.0694919	0.0208476
	$\hat{\lambda}$	2.489531	0.754014	0.09763	0.0001096	0.0209383	0.0052346

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